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ABSTRACT

This literature review explores the phenomenon of dysmathematics and problems in its definition. Dysmathematics is suggested as a modern term for low achievement in mathematics. This document addresses: (1) concomitant variables associated with dysmathematics; (2) mathematical characteristics of dysmathematicians; (3) diagnosis and prescription and their basis; (4) error patterns; (5) remediation of pupils with mathematical disabilities; and (6) didactics in mathematics for handicapped students. A theory describing dysmathematics as a set of complex and multi-factored disabilities is emphasized. Three major remediation models are presented: the content deviation model, the behavior deviation model, and the factor-interplay model, which proposes that mathematical learning should simultaneously comprise both subject matter and learner reactions. Further constructivist approaches to remediation are called for, with increased emphasis on play didactics. Includes 203 references. (PB)

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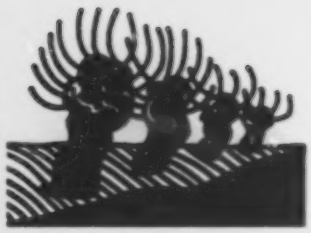

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Dysmathematics

Facts and theories concerning mathematics learning for handicapped pupils

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DYSMATHEMATICS

Facts and theories concerning mathematics learning for handicapped pupils

Olof Magne

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Dysmathematics (Dysmathematica) is suggested as a modern term for low achievement in mathematics in the compulsory school, in medical practice and other relevant conditions, since nowadays the trend is to gradually turn mathematics into an integrated branch of learning. But knowledge is meagre with respect to mathematics learning of handicapped students, and particularly as to the mathematical competence of adult handicapped people. It is an intricate problem to define "dysmathematics", or disability to learn mathematics. To some extent, this is a consequence of the apparent shortage of research on the issue how the various handicapped children learn to solve mathematical problems and to get experience of numeration, geometry, algebraic topics and probability. The work also is mainly lacking on theories for mathematical remediation. Magne and Thörn have suggested a partially new theory based on their empirical error studies. Psychologically and medically dysmathematics appears as a set of complex and multi-factored disabilities. Research should be stimulated how handicapped individuals react to mathematics and how the units of mathematical contents can be arranged. Three main models are presented. The most recent one presupposes that mathematical learning ought to simultaneously comprise both subject matter and learner reactions. This approach is described as the factor-interplay model. New starting-points for remediation should be considered, for instance based on constructivistic theory. The author suggests that future school practice should emphasise play didactics more than is fashionable today.

Key-words: Constructivism, disability, dysmathematics, error pattern, handicap, mathematics learning, play didactics, remedial teaching.

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BACKGROUND CONSIDERATIONS

Historical notes

Difficulties to master the language have been recorded since long time and at least from the 18th century. One international term is dyslexia, denoting underachievement only in reading and/or spelling.

Discord between performance in mathematics and linguistic skills was reported comparatively late (Oppenheim 1886 and Brandenburg 1887). Works on neurological impairment in connection with calculation were published still later, for instance Peritz (1918) and Gerstmann (1924) in Germany and Henschen (1919, 1920) in Sweden.

At about the same time an interest for psychological sequels of mathematical low achievement emerged. Pioneers were Ranschburg (1905) in Hungary and Schonell (1937) in England.

Standardised tests appeared in arithmetic during the first decades of this century with innovations like those of Stone (1908) and Courtis (1909) in the United States. These attempts were followed up by error studies in abundance, mostly about calculation routines. Ranschburg was a precursor even here. Notable early studies are those by Phelbs (1913), Scott (1916), Staker (1917), Schmitt (1921), Weimer (1922) and Buswell & John (1926).

Research on remedial treatment and instructional didactics began even later and still is quite unusual, mostly due to lacking theory. Again Ranschburg (1916) ought to be mentioned as an initiator, together with isolated originators as Osburn (1924, 1929), Hildreth (1937) and Schonell (1937).

Early as well as more recent studies usually deal with low achievers in infant or primary schools. Very little is known of mathematics learning among disabled persons.

Up to this time there is no lack of publications on mathematical education for handicapped students. Since 1905 thousands of dissertations, manuals, bulletins and papers have been printed about disability to learn mathematics, although rarely investigated. Most publications confine themselves to a limited range of topics, such as

* calculation with natural numbers,
* aphasic phenomena,
* error studies restricted to simple calculation practices in the four rules with natural numbers and
* mathematics anxiety.

Research dealing with low achievers in mathematics has mainly been atheoretical and still seems to be lacking a theoretical basis. In this chapter relevant research in the remediation of handicapped students will be reviewed, with due consideration to the limitations of this research.

The chapter's major sections consider the terminology and its base, appropriate variables to investigate, characteristics of the disabled learner, models for remediation and potential fruitful programmes.

Terminology and its base

Ranschburg talked of arithmasthenia. Henschen suggested the term acalculia (indicating a total loss), and this term has often been used by neurologists. Gerstmann may have been the first author to use dyscalculia. This questionable word is composed by a Greek prefix and Latin main part and, therefore, seems doubtful from a linguistic point of view.

In addition, non-technical words are often employed, such as "slow learner in mathematics", "below average achiever", "low achiever", "student with learning difficulties in mathematics" or "failing student".

The terminology presupposes a theory of mathematics teaching and learning. Ranschburg, Henschen and many early writers shared the view that primarily the subject matter

constituted the criterium for assessing the achievement and that the relevant mathematical topic to assess would be simple mastering of numbers (operations with natural numbers, fractions, decimals and possibly also precentage), in other words elementary "school mathematics".

Since about 1950 the ambition has rised to constructing a well-balanced curriculum emphasising a broader mathematical framework than earlier, where broader content areas should be met by the learner in order to acquire both basic facts and relationships between content areas and content structures.

This new aim makes terms like arithmasthenia or dyscalculia deceptive.

Magne (1988) suggests that dysmathematica is an appropriate modern term for the low achievement in mathematics of a person who does not master one or several main areas in mathematics according to set standards. This refers to a given occasion or to a defined period of the learner's life, manifesting itself as a performance well below the standard of the age group of this person or below his/her own abilities. The low achievement may be a consequence of inadequate cognitive, affective, volitional, motor or sensory etc. development. Diagnosis and remediation, based upon this two-way criterium, would simultaneously consider both the mathematical topics and the individual's reactions to the subject matter.

This base for the terminology embraces all present classifications and definitions, for instance arithmasthenia, dyscalculia, Rechenschwäche, difficulté en calcul etc., and partial low achievement, such as "underachievement" only in mathematics. The latter phenomenon would include two alternatives: (1) Underachievement in relation to a general standard where the person displays low performance in one or several main areas in mathematics, compared with the standard of the age group, but otherwise is capable of average results. (2) The second alternative is performance below the level of the person's own ordinary ability.

Dysmathematical behaviour must be looked upon from a relativist view. The assessment depends on the learning criteria of the prevailing educational conditions. The standards may vary from one school system to another. In a Swedish inventory 1955 (Magne 1958) 12 % were considered as dysmathematics. The same investigation indicated that only 0.5 % could be assessed as underachievers according to alternative (1). In a U.S. Conference on "The Low Achiever in Mathematics" the participants defined "the lowest 30 % based upon achievement in school mathematics" as their target population (Woodby, 1965, p. 2). Research on criteria problems is seldom conducted.

CONCOMITANT VARIABLES ASSOCIATED WITH "DYSMATHEMATICS"

Neurological symptoms

The first publication on the neuropathology of arithmetic, known to the author, was published by the German physician Peritz in 1918. In this work Peritz discusses the symptoms among soldiers who had been shot in the back of their heads.

Peritz reported a disturbed ability to perform calculations in case of leftsided but not rightsided injury in the back of the head. He suggested the existence of a number centre in the angylar gyrus. In addition, he assumed a close correlation between arithmetic and optical functions.

Henschen's research (1919, 1920) is considered the most important pioneer work in this field. Henschen made a thorough investigation of patients with a loss of the arithmetic skill in order to locate various forms of "acalculia". His research comprised own anatomic discoveries and literature analyses. Henschen has carefully studied 260 patients. Obviously, his material is very extensive from a quantitative point of view. His clinical cases are, however, collected from a great many sources, often incomplete report accounts in the literature and these vary as to the number of recorded data and the quality of the descriptions.

Henschen's investigation resulted in the following main conclusions:

(1) Linguistic and arithmetical types of aphasia usually are

concomitant.

(2) In a considerable number of the cases acalculia is non concomitant with linguistic aphasias.

(3) The extent of an aphasia, including acalculia, is directly proportional to the brain lesion.

According to Henschen there are one or more number or arithmetic centres which are wholly or partially separate from language or speech centres. Henschen tried to locate all observed arithmetical aphasia varieties, such as number deafness, number blindness, number aphemia (i.e. inability to pronounce numbers), number agraphia, paracalculia (i.e. mistakes at choice of rules of arithmetic) and amnestic acalculia (i.e. inability to name number facts). It seemed likely to Henschen that a main arithmetic centre should be found in the left occipital cortex. Subordinate centres might exist in other parts of the brain, for instance in the parietal, temporal or possibly central lobes, representing reading/writing of numbers and mental calculation. On the whole, Henschen stressed the importance of the optic brain functions, thus supporting the view of Peritz.

However, there seems to be no single circumscript mathematics centre.

Some neuropsychological studies seem to support this theory. Rourke & Strang (1983) maintain that mathematical underachievers perform poorly on measures of visual-spatial and visual-perceptual abilities.

Another series of ingenious studies are those from Gerstmann (1924, 1940) who has written several papers about the syndrome named after the author (the Gerstmann syndrome). It has been regarded as a parietal-occipital syndrome, with some resemblances to the Henschen-Peritz acalculias. As his complaints were of minor degree of difficulty Gerstmann used the term dyscalculia. He locates a centre to the dominant hemisphere, particularly in the part that is represented by the transitional region of the angular and the middle-occipital convolution.

Critchley (1953) has produced an excellent bibliography of the literature of this syndrome. It is characterised by four main symptom groups, namely dysmathematical behaviour,

finger agnosia (difficulty to name the fingers), dysgraphia (disturbance in the execution of letters or words although the patient is able to copy) and right-left disorientation.

The studies of these authors have been followed up without any tangible additions or improvements by a rather small number of studies, for instance Critchley (1953), Cohn (1961, 1968), Luria (1969), Johnson & Myklebust (1967), Perret (1973), Košč (1974), Hrbek (1976) and Collignon et al. (1977); see also Lezak (1983).

Later neurological research has yielded very few additional contributions to the study of mathematical psychopathology.

Bawkin & Bawkin (1960), Cohn (1968), Košč (1974)have introduced the expression "developmental dyscalculia", possibly influenced by Soviet defectology. They refer to organic brain disorganisation in the growth of mathematical representations, due to genetic disposition or lesions to the brain.

The discovery of hemispheric specialisation may be applicable to the theory of treatment of dysmathematics students in the future although there are no certain indications so far.

Wheatley et al. (1978) maintain that there is a division of labour between the two hemispheres in mathematical problem solving. The authors assume that the left hemisphere mainly is responsible for reading, reasoning and traditional school mathematics. The right hemisphere is better at geometric performance, recognition of wholes but is rather inactive in formal logic. Creswell, Gifford & Huffman (1988) confirm this view.

Wheatley has observed that the right hemisphere is active in concrete problem solving situations but that the left part of the brain takes over in problem solving without concrete material.

Some researchers (e.g. Luria 1966) do not seem to agree with Wheatley on the right-left-sided localisation of mathematics. He and others report of lesions in the right hemisphere, correlated with traditional mathematics.

Still another phenomenon is worth mentioning. Grissemann & Weber (1989) refer to the so-called MBD syndrome (minimal brain dysfunction) as a possible disturbing factor. They have

presented the following hypothetical ethiological subcategories.

(1) Exogeneous cerebral dysfunction of the MBD
   a. caused by a mild lesion in the early childhood
   b. caused by sensory deprivation and
   c. a combination between mild lesion (a) and sensory deprivation (b).
(2) Genetic disposition leading to minimal brain dysfunction.
(3) Sequels of undeterminable nervous disorders.
(4) Accumulations of conditions referred to above.

However, there is very little known how the MBD syndrome affects mathematical achievement.

Grewel (1952, 1969) has summarised the research results and suggests that aphasic disturbances in the field of mathematical achievement might be attributed to three principal categories.

(1) An occipital type (Peritz-Henschen acalculia) which he finds to be the outcome of severe occipital lesions.
(2) A frontal type which he refers to a general lowering of the intellectual level.
(3) A parietal type with concomitant reduction of writing, reading and memory components (Gerstmann syndrome, some of of Henschen's and Peritz' cases).

Psychological characteristics of "dysmathematical learners"

Since 1950 Magne conducted projects in Sweden to get information how lowachievers in mathematics react when they learn mathematics (Magne 1973). Other psychological analyses have been published by Ranschburg (1916), Schonell & Schonell (1957), Hildreth (1947), Fernald (1943), Ross (1964) and Jaulin-Mannoni (1979).

In the Swedish studies more than twenty thousand pupils and thousands of teachers have participated. Problems that have been investigated include the following,

* What are the typical reaction patterns of lowachievers ?
* Are there errors or thinking patterns peculiar to them?
* How can we increase the efficiency of special education, ordinary classes and mathematics clinics?
* Can we find a rationale for diagnosis and learning/teaching?

The Swedish results indicate that low achievement in mathematics appeared as very common in most school systems.

About fifteen per cent of the Swedish pupils in compulsory education are affected and even more in the secondary schools and the special schools.

Four main symptom groups stand out as the most frequent ones among low-achievers:

* Various forms of learning disability, such as low intelligence, a difficulty to form new associations, low abstraction capacity, and a low learning capacity (more than 95 % of the lowachievers).
* Reduced staying power or will power, such as day-dreaming or impaired orientation reactions (cp. Galperin, 1974) (more than 75 %).
* Affective disturbances, often specificly linked up with mathematics, such as specific dislike of mathematics or specific mathematics anxiety (25 to 50 %).
* Instability, hyperactivity, restlessness or reduced concentration (about 50 %).

The researchers are usually not able to determine the causes of dysmathematics. The following conditions were notable in the Swedish studies:

* There were more brain injuries among lowachievers in mathematics than in a random group of children of the same age. The proportion of brain-injured lowachievers did not exceed 20 percent. This means the overwhelming majority of all students with dysmathematics could not be shown to have neural dysfunctions.
* Only a few lowachievers have parents who explain that they also had difficulties to learn mathematics when they went to school. Among the dysmathematicians the following was noted. There was a significant difference between low-achievers and controls. This does not indicate that the difficulty was hereditary. It seemed to the interviewers that some parents had influenced their children to feel insecure about mathematics and this may have induced the pupils to low achievement.

  In the light of this circumstance it seems extremely doubtful whether the syndrome of developmental "dyscalculia" (see for instance Košč 1974) exists but in very few cases.
* According to many studies (cp. Woodby, 1965) many low-achievers belong to disadvantaged families. This may be the case in many countries. In Sweden (Magne, 1973) and

Switzerland (Grissemann & Weber, 1982) where this problem has been carefully studied poverty per se does not seem to be a principal cause of low achievement in mathematics. Reading and writing is suggested to be more affected. An explanation (Rosner 1956) might be that destitute parents convey to their children the value of money even if they do not entice the children into reading and writing.

* Behaviour disorders usually seemed to cause the low achievement in mathematics. These seem to fall into the following categories,
  - The inertia pattern, indicating a working disposition where the pupil displays passivity, day-dreaming, reticency, idleness and lack of mental vigour.
  - The withdrawal pattern, manifesting itself in low sociometric assessment in various categories.
  - The immaturity pattern where both parents and teachers observed that the children had generally shown a slow development or maturity and seemed to behave as younger children.
  - disturbed work habits, involving lack of interest,home or school maladjustment, short attention span or limited initiative.

  In tne Swedish investigations there were only exceptional cases where the school appeared to have provoked low achievement. Thus, there were registered various symptoms of maladjustment before and after starting school. For most of these children the number of symptoms decreased from the age when the children began school.

There is also a specific learning difficulty (or under-achievement) in all types of school systems, characteristic to each. Specific difficulty is defined as achievement falling below a certain criterium for other school subjects (see definition above).

Specific learning difficulties ("specific dysmathematics") are exceptional, less than one percent of the population in the compulsory school, including pupils in special schools.

Also in special schools there are pupils with specific learning difficulties in mathematics, that is lower achieve-

ment in mathematics than in other school subjects.

The correlation between various personality traits and performance in mathematics has been examined. These studies seem to agree that many underachieving persons have emotional and personal-social disorders. Such personality disturbances are much more frequent among underachievers than low-achievers. The disorders tend to cause the learning difficulties. One of the more remarkable findings is that usually these problems depend on conditions outside the school.

Summarising these results, mathematical low-achievement and underachievement appear as complex and multi-factored disabilities. These children ought to be treated not only with respect to the learning difficulties in mathematics but, in addition, to several other handicaps as well.

It is, by no means, enough to train computational skills. A good balance between main topic areas proves to be useful.

Many children need a psychological therapy also, in some cases a deep-going psychotherapy.

Many researchers have found it important to individualise the remedial programmes for the pupils with dysmathematics. There was no single or easy way to remedial teaching or successful learning.

This means that mathematics learning/teaching has two necessary dimensions: topic and behaviour. We should not exclude one or the other of these two dimensions in our treatment of the child with learning difficulties.

Nevertheless, in some schools or school systems the curriculum seems to miss this requirement, perhaps not totally but to some extent. Some curricula have tended to pay too much attention to the mathematical subject matter without realising the behaviour development of the individual learners. On the other hand, there are also curricula which might be stressing too much the importance of psychological factors or social experiences of the pupil.

MATHEMATICAL CHARACTERISTICS OF DYSMATHEMATICIANS

Under-achievement in the ordinary school population

A common tacit understanding among researchers is to treat

pupils of about 7 to 15 years of age as dysmathematic if their average mathematical performance is rated at two years below their mental age.

Bibliographies of mathematical underachievement are found for instance in Fleischner & Garnett (1980), Matulis (1981), Lorenz (1982), Fletcher & Loveland (1986), Bley (1987), Grissemann & Weber (1989), König (1989) and Magne (1989).

The accuracy in some main areas of mathematics of low achievers, compared with average pupils is demonstrated in table 1 from Magne & Adrell (1973). The subjects in this investigation were pupils in Swedish compulsory education aged 8 to 13 where two groups were underachievers, that is disabled only in mathematics (E_1 and E_2), and one group of average achievers (C). E_1 and C were matched groups from identical conditions, educationally and socially (except in mathematics). E_1 was given remedial teaching, but not E_2. It is evident from table 1 that the mathematical knowledge of the underachievers is much lower than that of the average pupils.It should be observed that the average pupils had quite high frequences of occasional failures.

The pupils in the underachieving groups had a general intelligence that should have enabled them to learn the elementary items of the testing programme nearly one-hundred-per cent. This was not the case. They were hampered by insufficient experiences and inadequate mental dispositions. It is a plausible supposition that due to early delayed development they had missed basic learning opportunities in the previous grades. Their learning had not built upon their proper qualifications and, besides, proceeded with more and more difficult topics based on a knowledge level they never had reached.

The differences between the underachievers and the average pupils were nearly always statistically significant. Also for topics where all groups displayed few difficulties the differences were fair-sized, for instance conception of expressions as greater than, less than etc., understanding of two-digit natural numbers, combinations of units and common denominate expressions, such as money, length, weight, and volume.

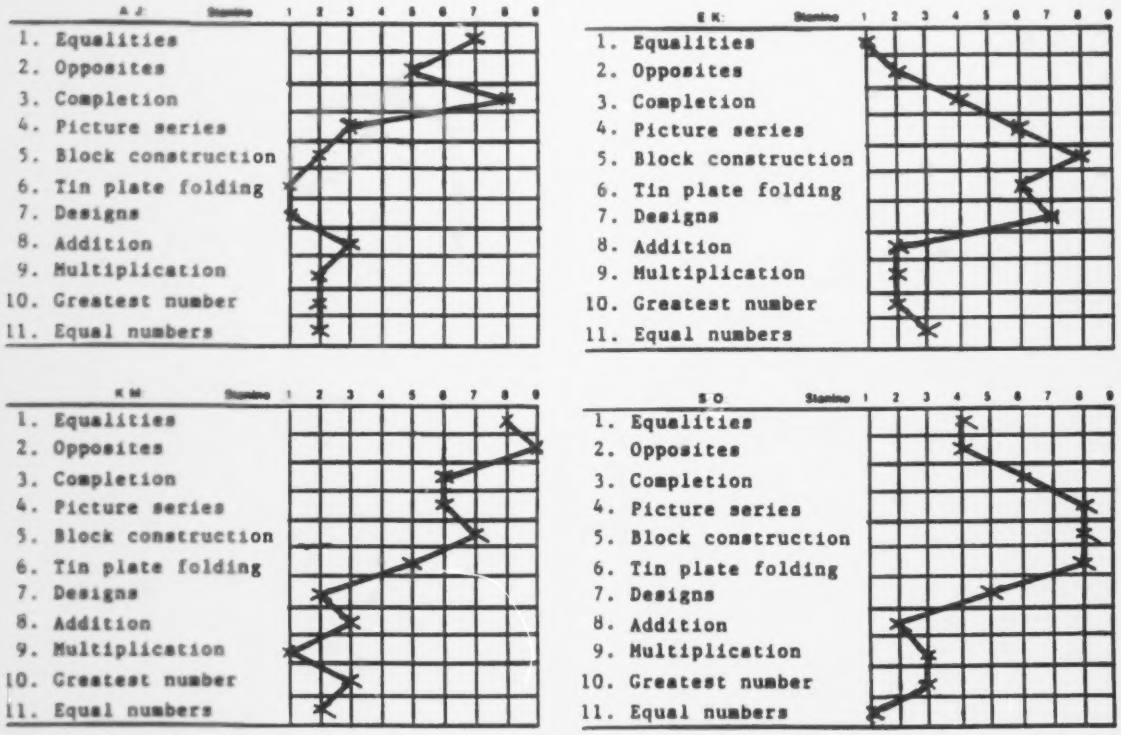
Table 1

Mean percentage assessed performance in some main areas based on the Magne Mathematics Diagnostics Tests in three groups of underachievers (8 to 13 years of age).

Mathematical main areas	Percentage of pupils mastering the main areas before remediation		
	E_1 N=125	E_2 N=104	C N=125
Numeration, notation	29 %	32 %	62 %
Basic processes of computation	26 %	17 %	56 %
Measuring, geometry etc.	6 %	13 %	50 %
Verbal problem solving	3 %	9 %	46 %

Table 2

Individual profiles of underachievers in mathematics in the Differential Aptitude Test (from Magne 1967).



The conclusion is that no topic exists where under-achievers ordinarily keep on level with higher achieving pupils. Another important finding is that the attainments differ not only quantatively but also qualitatively. This phenomenon will be further analysed.

Some underachieving pupils do not reach the same performance level in all mathematical main areas.

Some findings of another Swedish survey illustrates this (Magne 1967).

Underachievers were assessed with the help of a differential aptitude test. The subtests 1-3 were verbal logical, 4-7 non-verbal/spatial and 8-11 had a mathematical structure.

For the whole sample the means were low in the subtests 8-11. See table 2. A matched control group had average results in the subtests 8-11.

It is interesting to interpret the profiles in table 2. They give clear and distinct evidence of variations between the underachievers, but also of variations within the individuals. The average learning aptitude is just a little lower than the mean of the control group, but the pupils may have above-average or below-average scores in some test categories.

The low mathematical attainment of pupil AJ is probably partly explained by his weak spatial aptitude. Similarly, the underachievement of pupil EK may depend on his low verbal capacity. The other two cases (pupils KM and SO) seem to be specific dysmathematicians, in the sense: handicapped only in mathematics, the poor achievement exclusively caused by their low mathematical ability.

It follows from these short notes that AJ is weak in geometry and arithmetic, but comparatively capable in verbal problem solving. Also KM has fairly good results in problem solving. SO is poor in all areas except geometry. EK is weak in all areas.

In order to explain cases like these, Frostig & Horne (1964), Hoffer (1977), Krutetskii (1976) and Magne & Thörn (1987) refer to their research on verbal-logical and visualisation factors. See also Grissemann & Weber (1989).

What to do in remedial mathematics if the student has poor visual perception? The preparations may be different if the

verbal-logical capacity is high, medium or low.

We might argue that high verbal-logical ability poses few problems.

Low verbal-logical ability leads to difficulties if the teacher does not find ways to help the child to make the best use of his gift to shape clear visual images.

Medium verbal-logical ability combined with low visual images may not be too debilitating. However, weakness in visual perception often causes considerable difficulties in mathematics if the teacher is not observant.

On the whole, little is known about the causality of under-achievement. Gruszczyk-Kolczyńska & Semadeni (1990) and Haylock (1986) point out that underachievers usually display a history of dysmathematics already before school beginning. Thus, immaturity in learning mathematics affects later development.

"Number anxiety"

There is a considerable amount of literature on attitudes and other affective manifestations associated with mathematics learning. Biographies have been published by Biggs (1959, 1962), Brown & Abell (1965) and Magne (1964, 1973).

Dutton (1956) made a representative study of teen-aged students in California were only 16 % strongly disfavoured mathematics, a result that Callahan (1971) could support. Several recent studies agree that the subject's popularity decreases from the first to the last school year.

Slade & Russell (1971) found that children with primary emotional disturbances often have outstanding difficulties in arithmetic.

Hanfman (1955) discovered that arithmetic test items were serviceable as psychodiagnostic instruments in order to measure anxiety. See also McLeod & Adams (1989).

The role of anxiety is complex. Some degree of anxiety is profitable and raises mathematical attainment. But strong anxiety, particularly specificly tied to mathematics seems to have negative consequences. Pupils differ as to their degree of anxiety as well as to its consequences. Castaneda, Palermo & McCandless (1956) suggest that highly anxious pupils learn easy topics faster than low-anxious pupils, but low-anxious pupils learn difficult topics better than

highly anxious pupils.

Magne (1964, 1973) made the inference that

* mathematics arouses more negative stress than any other school subject,
* this stress/anxiety is especially frequent among lowachievers, and
* in these cases stress/anxiety causes learning difficulties in mathematics.

Obviously, anxiety may manifest itself in many ways. There are two main interpretations of the nature of anxiety which is connected to mathematics. One opinion is that there are several types of anxiety (Biggs 1962, Spielberger & Powell 1965, and Trondahl & Powell 1965), But, in addition, a specific anxiety has been found which is specificly tied only to mathematics (Magne 1967, Fennema & Sherman 1976, Frary & Ling 1983, and Sandman 1980).

According to both theories anxiety develops through constant failing. There is no escaping one's fate namely to be influenced from constant disillusionment at failures during the schooling. From the beginning the pupil expects to learn mathematics successfully. But if success does not occur the self-esteem and zeal of the lowachiever wanes. Various symptoms follow, as aversion, anxiety, aggression and attempt to escape. In severe cases the strength of the feelings justifies that we talk about panic (Buxton 1981).

Mentally handicapped

Research in mathematics education of the mentally handicapped has as long a history as research on dysmathematics (Ranschburg 1905). Later significant studies include Cruickshank (1946, 1948), Klausmeier, Feldhusen & Check (1959), Kirk (1964), Bleidick (1966), Bower (1970), Conolly on KeyMath (1968,1971), Conolly (1973), Reisman & Kaufman (1980), Gelman (1982), Baroody (1986, 1988), and O'Toole & O'Toole (1989). Of great interest are volumes 1, 9 and 10 of the series Soviet studies in the psychology of learning and teaching mathematics. Important studies emanate from Polish universities, e.g. Gruszczyk-Kolczyńska, and Karwot, Warsaw, and Wyczesany, and Siwek, Cracow. Other relevant studies include equivalence or

inequivalence conservation problems according to the Piaget technique, such as Inhelder (1944), Woodward (1959), Lovell & Slater (1960) and Lunzer (1960).

As a group mentally handicapped individuals are in every respect very much inferior to average individuals in all areas of mathematics. Several studies have compared the mathematical attainment of mentally retarded children with other children of the same mental age. Even by this method, the attainment of mentally handicapped often is inferior to the attainment of a control group, for instance on topics as counting, understanding of numbers, notation, and even place value, addition, subtraction, numerical reasoning, most measurement (as temperature, length, weight, volume, area), form perception, body awareness, geometry, verbal problems, functions and probability.

Mentally handicapped, after all much older than the controls, can be found significantly superior in some tasks of simple multiplication and division (e.g. tables) and facts on money and time (e.g. calendar). The superiority may be attributed to more years of schooling.

For mildly handicapped Bleidick (1966) has found an average retardation of four years at the age of 16.

However, it is interesting to note that among mentally handicapped children and adult persons there are a few "idiots savants" who possess a paradoxical skill in "number work", for instance to multiply numbers running into several figures.

Mentally handicapped obviously display definite failing in all areas of mathematics that require verbal-logical mediation. Connolly (1973) strongly stresses their weakness in work habits, difficulty in following or understanding directions and inadequacy to organise their learning systematically on their own. Cruickshank (1948) refers to their disability to isolate pertinent information from a body of given data and to organise, process and make use of this information, cp. also Krutetskii 1976, Kalinykova (1975), Mikhal'skii (1975) and Tishin (1975).

Connolly also suggests that instruction for mentally handicapped should be individualised and tailored to the

particular aptitudes of each child. The teacher should provide the mentally handicapped child with an opportunity to gain greater precision in his thinking and his interrelating real-life experiences with the language of mathematics, often with the help of unconventional learning methods.

As Cawley & Goodman (1968) have pointed out there exist a number of experimental possibilities in the mathematical programming for the mentally handicapped. One research approach might be to analyse the mathematical development among the retarded. A second approach is to inquire into the processes to solve verbal and practical problems. The programme activites might focus more attention on problem types where the children can construct the solution with the help of materials. Another intervention technique would be to include pretraining activities. This may lead to meta-cognitive activities, or learning-to-learn skills. A fourth possibility - still scantily scantily explored - is to use play didactics with handicapped.

Neurologically handicapped children

Haskell, Barrett & Taylor (1977) point out that the development of arithmetical skills has attracted little attention in research on children with neurological handicaps. Among the few explorations of mathematical learning, Abercrombie (1964) has found indications that cerebral palsied children are highly vulnerable to arithmetical failures. Phillips & White (1964) have noted that motor-disabled children with brain injuries have significantly lower arithmetic performance than physically handicapped youngsters without neurological lesions. Henderson (1961) maintains that more than 90 per cent are backward in mathematics compared with average children. Kaliski (1962) explains this backwardness with reference to verbal shortcoming. Another reason might be that the present mathematics education is inefficient and, in addition, that this is an international fact (Guttman 1990). According to her research there exists no comprehensive knowledge of the mathematical performance or learning among the various neurologically handicapped children.

Hearing impaired children

Until recently very little was known about mathematical attainment of hearing impaired children. Hitch, Arnold & Phillips (1983) have studied counting habits in deaf children compared with hearing controls. They found the same counting processes in both groups. Worsley (1965) considered deaf children impaired in all four basic operations of arithmetic. Jensema (1975) and Wolk, Karchmer & Schildroth (1982) mean that general ability is a decisive factor. Karchmer, Milone & Wolk (1979) point to the educational significance of the degree of the hearing loss. Race and family income constitute other important symptom groups.

Wood et al. (1983) reached the conclusion that deaf school-leavers are typically retarded by about three years relative hearing children. But the degree of hearing loss was not a determinant of performance. Wood et al. (1984) studied hearing and hearing-impaired pupils attending ordinary schools. Multiple regression analysis revealed that educational placement per se was no important determinant of achievement in mathematics.

No adequate explanation has been found why hearing-impaired children do less well than their hearing peers. A possible interpretation is that didactical routines are not efficient. This is also stressed very strongly by Kluwin & Moores (1989).

But in addition, Bodner-Johnson (1986) has suggested that deaf students with high mathematics achievement had families who expressed high educational and occupational expectations of standards for their children.

The causal relations may also differ for deaf and partially hearing persons.

Since a long time the controversy between didactics based on oral communication, on one hand, and sign-language or total communication didactics on the other hand has dramatically interfered with practical efforts to organise a balanced mathematics education for hearing-impaired children.

Bödör (1979) refers to research and development work in Hungary where this controversy has been discussed. According to Bödör Hungarian studies indicate that it is not sufficient

with "quantitative increase of concrete-practical activities alone" (p. 248). He argues that first of all the communication functions of speech should be developed in remediation. But Chasen & Zuckerman (1976) found that a total communication class improved more in mathematics than an oral class.

This may not be the whole truth although Bödör certainly is right about the need of comprehensive balanced didactics for hearing impaired. Reasearch is lacking how such learning should be realised.

Sight impaired children

There seems to be very little research on mathematical behaviour of visually impaired indivduals. A bibliography is presented by Ostad (1988).

There are some studies of conservation (Piaget), for instance by Gottesman (1973) and Miller (1969).

The usual theme is computation of natural numbers. According to some investigators the attainments of congenitally blind children shows a "lag of development" up to two years in acquiring specific knowledge (Hatwell 1966). Nolan says (1964, p. 97), "For decades, mathematics has been a source of major frustration for blind children and their teachers." The same opinion has been expressed through the findings of Nolan & Ashcroft (1959), Hayes (1941) and Brothers (1972). These investigations agree that the blind children are below the mathematical achievement level of their sighted peers by about 15-27 per cent, as measured by standardised achievement tests. Ozias (1975) is surprised that the low attainments of visually handicapped children has not improved more considering the efforts during the last decades to improve education in mathematics for the visually impaired.

Blind children may, however, experience their most serious difficulties in areas that may be particularly significant for their mathematics learning, such as spatial orientation, classification and class inclusion (Stephens & Simpkins 1974). Psychological and educational investigations demonstrate that the blind children constitute an extremely heterogeneous group. According to Warren (1984) studies of

visually impaired children do not always pay attention to the fact that test items presented to both blind and seeing children may have unlike meanings to the two groups. Ragosta & Kaplan (1986) and Packer (1987) have demonstrated that the visually handicapped need much longer testing time on the SAT than ordinary pupils or other handicapped. The testing times are the following for some handicap groups:

Hearing impaired	average	3.7 hrs.
Visually impaired		5.4 hrs. (casette version)
		6.0 hrs. (braille version)
Physically disabled		4.3 hrs.
Learning disabled		4.7 hrs.

The observed lag of development of mathematical achievement has been explained partly as a consequence of the visual impairment itself, partly as a result of concomitant lack of experience (Warren 1984).

DIAGNOSIS, PRESCRIPTION AND THEIR BASIS

It is necessary to proceed from atomistic or piecemeal error studies to more wholistic work and, thus, implement efficient remedial practices. But also in this case it seems advisable with theoretical foundation.

A cognitive theory for the elementary mathematics education

Early diagnosis and intervention were based upon an associationistic viewpoint both in Europe and America. Criticism from the gestalt psychologists in Germany and the American functionalists brought up a more comprehensive, motivational conception. The French and British cognitivists, such as Wallon, Piaget, Bachelard, Bartlett and Skemp lead up to the modern constructivism. Constructivism stresses that learning is a private affair, a continous restructuring of the learner's prevailing thinking structures (Piaget: "Schemes"). The role of the educator is to transmit information to be used as a source for the pupil's knowledge. Many researchers

have accentuated that active learning is a necessary condition for lasting mathematical acquisition.

Rourke & Strang (1983) warn that in present research strategy it is a failure to miss carefully defined criteria in selecting mathematically disabled pupils for study. If too heterogeneous groups of children with mathematical problems are examined, the results may indeed serve to becloud rather than to elucidate the processes of various subtypes of dysmathematic pupils.

Magne and Thörn (1987) have recently presented a report, called "A cognitive taxonomy for mathematics teaching". Cognition has to do with knowledge and thinking. Taxonomy means that we classify topics of mathematics teaching according to their natural relationships. The taxonomy of Magne and Thörn may be defined as consisting partly of a learning and teaching system, partly of a category system.

This is a theoretical analysis based on empirical error explorations of more than one million operations. The error studies firstly resulted in an empirically developed category system and secondly was extended to the resulting system of learning and teaching elementary mathematics.

For the Swedish compulsory school (7 to 16 years) Magne and Thörn presented six main areas in mathematics education which certainly are interrelated in a factor-analytic sense. They are constructed with reference to factor patterns of the mathematical ability. The main areas are

P-area: Linguistic representation and problem solving,
N-area: Numeration and notation,
G-area: Form perception and space representation, body awareness, money, geometry, measuring and units,
ASMD-area: The four operations,
F-area: Functions, algebra, equations, and
B-area: Descripitve statistics, probability.

The main areas are divided up into content/pupil reaction subareas. They are called categories. A category is regarded as a function of subject matter (stimulus) and pupil reaction (response) and is conceived as an expression of thought and action processes. The third (primary) level is built up by subcategories. This means that each category consists of entities signifying behaviour learning units.

The cognitive taxonomy started with Magne's studies (1958) on remedial mathematics for school children who made many mathematical errors ("dysmathematicians"). The present project began 15 years ago and continues. It has already been used for several practical and theoretical purposes, including mathematical remediation, such as

* Improving diagnosis
* Preventing learning difficulties, and
* Constructing programmes for remedial teaching.

A case study. The diagnosis and learning programme of "Nils"

Diagnostic practices have been worked out from the cognitive taxonomy. This work includes tests, interviews and observation, assessment techniques and evaluation methods.

The planning is illustrated by the flow chart in fig. 4. In a class of ten-year old children the mathematical abilities were tested in August at the beginning of the school year (Grade 4 of the Swedish elementary school). With the help of these tests and additional individual interviews and observations we assessed the mathematical knowledge of the pupils and followed up with the needed behaviour assessment for dysmathematical individuals.

In this procedure we talked about quantitative and qualitative diagnosis.

The quantative diagnosis relates to statistical standards, in this case stanine scores. "Nils" in table 3 is compared with average children of his own chronological age or with younger children who have the same test scores. The scores of Nils indicate that he is about two years behind his age group.

The qualitative diagnosis is founded on the Magne-Thörn category system. The test results of Nils are interpreted from a differentiated assessment of his solutions of the various test items. The qualitative diagnosis of Nils suggests that he should work with mathematical topics of an elementary stage and tnat his performance is low in all reported four areas.

Table 3. Diagnostic survey in mathematics topics for "Nils". Age: 10 yrs. 4 mo.

Quantitative diagnosis: "Magne's "Diagnosis in mathematics No. 12, 13, 14 and 20

Test:	Score:	Stanines:	Assessment:
12: all main areas	24 out of 27	5 for 8 yrs. 3 for 9 yrs.	Attainment comparable with
13: all main areas	13 out of 22	3 for 8 yrs. 2 for 9 yrs.	the average for 8 year old
14: all main areas	9 out of 28	3 for 9 yrs.	children.
20: addition+subtraction	8 out of 50	2 for 10 yrs. 2 for 9 yrs. 1 for 10 yrs.	

Qualitative diagnosis:

P: Linguistic representation and problem solving.
Solves simple text problems and oral problems.
When the wording is more complex, various errors appear.
Linguistic analysis and vocabulary are insufficient so he has difficulties to grasp subtle semantic distinctions.
Essentially there is a lack of practical experiences which are to be transformed into linguistic interpretation or number conception and space representation.

N: Numeration and notation:

Nils has great difficulties with his conception two-digit and three-digit numbers.
He manages to write numerals for natural numbers 0-100.
Three-digit numbers are often analysed incorrectly.
He only writes expanded numbers of natural numbers 0-100.
He finds it difficult to explain how number series are built up.
Nils often misses the point of regroupings in arithmetical algorithms or conversion of units.

G: Form perception and space representation, body awareness, money, geometry, measuring and units.

Basic form perception is satisfactory. Also motor skills.
Knowledge of money/change is insignificant.
This also is the case with measuring and weighing, for instance length.
Unreliable in attempting to solve tasks of compound calculation and reduction.

ASMD: The four arithmetical operations.

Nils has forgotten much about addition and subtraction.
Multiplication and division: forgotten nearly everything.
"Diagnosis 20" presents a variety of unique procedures, particularly in subtraction, probably well consolidated.
Makes errors even in calculation with one-digit numbers.
In regard to this, relearning is necessary.
The error patterns seem to be due to the fact that Nils has misleading concrete experiences concerning numeration and linguistic representation.

Other behaviour observations.

Nils has an IQ of about 90. No observed personality disorders.

In his qualitative analysis of curriculum tasks the diagnostician moves through the categories of the taxonomy and registers the mathematical learning prerequisites of the pupil. At the same time the general behaviour is examined. The diagnostician assesses the pupil's behaviour status as accurately as possible.

From this background the diagnostician works out a learning programme together with the teacher, the parents and the child.

Other forms of diagnosis

I have concentrated this survey on the diagnosis and learning programmes of the children. But this is not enough. There are other aspects of diagnosis which could and should be implemented in the class.

Diagnosis and prescription ought to have additional aims. We should test and diagnose the instructional materials in order to find those which give optimal learning to each class. It is important to find the best set of instructional aids. Just this class we are talking about had been assigned a textbook and an exercisebook which did not suit the class very well. Our diagnosis of the available material had the consequence that some exercises were excluded and additional materials and exercises were included.

We also diagnosed the children's need for instruction, with the help of the cognitive taxonomy. Many children had difficulties to remember the topics. They needed many repetitions and distributed learning. This led us to the use of the spiral technique in a particular form: The teacher brought the various topics forward just so far as was optimal for the respective groups of pupils and at this moment another topic would be introduced. With the help of this approach the teacher could better individualise the education, particularly for the lowachieving pupils.

ERROR PATTERNS

From psychological aspect, Magne distinguishes between only two major categories:

* Errrors due to incorrect or incomplete logical reasoning, and
* Errors due to oversight or perceptual interference.

Comparisons between high-achievers and low-achievers

In our Swedish error analyses inventories were made concerning several thousand children's computations, problem work, geometry, equations etc., including interviews on thinking techniques, partly from systematic testing in schools, partly from ordinary lessons. In some studies we used contrast comparisons between highachievers and lowachievers, aged 7 to 16 years. The contrast groups consisted of various proportions of the age groups at issue, from 5 % to 27 %.

First, similarities.

These comparisons show that lowachievers display many similar error patterns as the average or above-average pupils:

* The first thing common to all pupils is that they do both correct and incorrect mathematics because they think.
* Secondly, it seems obvious that many errors are caused by oversight in all pupils, or an accidental lack of concentration.
* The third important observation is that all pupils are working with an apparent striving after meaning.
* Fourthly, it is evident that lowachievers and underachievers, as well as other pupils, form structures and do their problem solving or computations in accordance with "schemes" or strategies they get from earlier experiences.
* Fifthly, errors can be looked upon as symptoms of individual shortcomings in the interplay of mathematical contents and the pupil's personality.

No less important is the finding that in some respects error patterns are very much dissimilar for highachievers and low-achievers. The following three observations seem to be particularly remarkable:

* Most errors of lowachievers are errors of lacking or insufficient logic. Most errors of highachievers are caused by oversight.
* Lowachievers not only make systematic errors but, if not given remedial intensive intervention, continue with the same error pattern for long periods of time. Highachievers more seldom make systematic errors and they easily give up error patterns.

* Remediation is a comparatively easy process for highachievers whereas it is an extremely time-consuming and laborious affair for lowachievers.

Krutetskii (1976) has conducted painstaking analyses of individual students' thought processes by means of contrast studies. He observed that mathematical achievement may depend on rather dissimilar psychological connections in various pupils. In his contrast comparisons he remarks that there are typical differences between a highachiever and a lowachiever in three central respects:

* Information gathering, including initial orientation to a problem,
* Information processing during problem solving situations, and
* Retention of the information material.

It has been maintained that error patterns are about the same in all pupils (Cox 1973). According to our findings this view is incorrect (Magne 1989, Krutetskii 1976). Thus, average and above average pupils may display other types of errors than lowachievers. Also error profiles can be different for individual pupils (Lorenz 1982, Magne 1988a). Compare also results of Janke's studies (1980) of computational errors of mentally handicapped children.

Causes of mathematical errors

Few reliable specified investigations exist. The attempts to study causes of errors are formulated in far too general terms, for instance the classifications of Radatz (1980), Gerster (1982) and Lorenz (1982). Lorenz suggests the following causality: developmentally conditioned error patterns and error strategies due to misconceived information gathering or information processing. He seems wrongly to assume that oversight errors are infrequent. Lorenz considers underachievement as a separate cause of error patterns.

The work of a diagnostician may be compared with that of a detective in the Wimsey or Poirot tradition to deduce from seemingly insignificant clues what is going on in the child's mind. As we become better at this observation and detection technique our effectiveness as teachers will improve. With the

help of persistent, devoted diagnosis founded in modern remedial theory a trained diagnostician is able to prescribe a programme of good validity.

Error patterns connected with typical handicaps

We may hypothesise that children with typical handicaps present typical error patterns. But little is known.

Blind children are at a disadvantage because they have a handicap to visualise. Therefore, they may be inhibited in their efforts to imagine numbers and geometric concepts.

Deaf children may be linguistically impaired. This often leads to a restricted vocabulary and, secondarily, to a limited reasoning power.

Emotionally disturbed pupils may display a disordered state of mind, and their logical reasoning could become confused in such cases. Consequently, they may get difficulties to establish mathematical structures in problem solving.

Motor impairment often inhibits the growth of form perception and geometric notions.

A good deal information on mentally handicapped children indicates other mathematical strategies than those of the average pupils (see for instance Janke, 1980). The mentally retarded are able to acquire at least some knowledge inside all of the Magne-Thörn main areas. Individual differences are considerable.

Most of the retarded persons have a great difficulty in grasping the structure of the decimal system. Their mastery of mathematical expressions is usually extremely low (P-area), due to reasoning disability. This leads to inadequate strategies in problem solving. Experiences and conception of numbers (N-area) are often limited to two-digit natural numbers and very simple rational numbers. Simple visualisation and geometry skills (G-area) can be attained although more complex achievements may be unattainable. Of the four rules (ASMD-area) addition is better mastered than other arithmetical operations. Least skill is found in division. The F- and B-areas can usually not be managed. Desk calculators and even computers can be used in problem solving.

Assessment of error patterns by different research methods

Studies by Schonell & Schonell (1957), Magne (1958, 1973) and Kilborn (1979) indicate that teachers can very well decide the general mathematical ability but are unable to assess specific traits in dysmathematical pupils.

This leads to the recommendation that appropriate diagnostic instruments should be used whenever a pupil would be referred to remedial instruction.

Furthermore, observations by Krutetskii (1976) and Magne & Säll (1975) suggest that error studies ought to involve a variety of research approches. Written tests serve the purpose of taking stock of even unusual variants on a broad basis. Interviews has the purpose of finding individual methods used and errors made by children when confronted with tasks that uncover whether certain strategies have originated from the teacher or not. However, since the interview method is tedious and sometimes unreliable also observations should be used.

Trends in research of error patterns

Radatz (1980) surveyed error studies especially in Germany and USA. He suggests that during an initial period 1900 to 1930 German studies were characterised by efforts to find psychological explanations whereas in USA the typical research type was mapping out of error variants and their persistence. In both countries studies mostly dealt with simple calculation routines.

Current approach has been strongly influenced by cognitive learning theories, motivational explanations and reference to sociological research. Reference to mathematical structures is also more common nowadays. Calculation still attracts the chief interest. Although the balance theory for mathematical education has been stressed there are few studies being published outside the topic of simple calculation (the Magne-Thörn ASMD-area).

At the end of the 1980's Radatz proposes three main aspects characterising research on error patterns:

* Continued studies of forms and variations of errors,
* Error analyses as a diagnostic means to better teaching, particularly for dysmathematic pupils, and

* Studies to increase insights in the pupil's differentiated mathematical reasoning on consecutive stages of development.

There are some interesting innovations among recent studies. One example is Hart et al. (1981) which presents a broad survey of various topic areas. Magne & Thörn (1987) have used empirical explorations in order to build up theoretical structures. Garfield & Ahlgren have analysed difficulties in learning basic ideas with regard to probability and statistics. Booth (1984) has made an inventory of algebra errors defined as "generalised arithmetic". Another promising survey is the 1988 NCTM Yearbook where several contributions take up students' difficulties in algebraic thinking (Coxford 1988).

A marked trend is formed by the attempts to integrate specified empirical findings into theoretical remedial models. Resnick's et al. (1989) is a good upholder of this trend with their documentation of errorful rules about decimals and fractions among school children in three countries.

A growing interest in the so-called "buggy algorithms" is noticeable in the sense the pupil's own errorful rules (cp. German: "Kinderfehler"). Clara Schmitt discovered the phenomenon in 1921. It was rediscovered several decades later. See Magne (1958), Lankford (1972), Cox (1975) and Ashlock (1976). These analyses show that "buggy algorithms" are intelligent constructions which are often built on incomplete rather than incorrect reasoning. They cannot be avoided by the pupil and may even be looked upon as useful diagnostic tools in the hands of the instructor. See also Movshovitz-Hadar, Zaslovsky & Shlomo (1987).

REMEDIATION OF PUPILS WITH MATHEMATICAL DISABILITIES

It has been suggested that constructivism is the basis for remedial work in mathematics. Through the individual's learning runs the connecting thought that the learner must begin with building up experiences. Next step is to discover invariances and to construct insights, generalised representations and abstract ideas. Then, the discovered topic structures are used to acquire reasoning patterns. A fourth learning phase has been characterised as skill acquisition.

Three guiding principles are mentioned by Strang & Rourke (1983):

* First, interventions should be as verbal as possible.
* Second, concrete and systematic instruction should be followed.
* Third, the dysmathematic pupils should be given opportunity to find out how to organise the knowledge. This means: structure is essential.

Promising practices are described in, among others, Woodby (1965), Ashlock (1972), Lowry (1972), Csocsán (1979), Jaulin-Mannoni (1979), Johnson (1979 a and b), Wittoch (1979-83), Glennon (1981), Matulis (1981), Grissemann & Weber (1989), Schöniger (1989) and Ostad (1990).

The content deviation model

The discussion about diagnostic and remedial education in mathematics is controversial. Three types of theoretical approaches have dominated. The oldest rationale can be described as the content deviation model. It dates back to associationism before the beginning of this century. The content deviation model focuses on mathematical content. In a simplified form it might be illustrated as in fig. 1. We consider, on one hand, various observed levels of mathematical complexity and, on the other hand the mathematical topics at issue.

The behaviour deviation model

About 1940 the behaviour deviation model was introduced. It relates to devious learning styles of the learner (for instance buggy algorithms). Fig. 2 is cited from Varga (1979). He particularly refers to handicapped pupils, such as physically, mentally, socially or emotionally handicapped, with possible overlappings, as shown on fig. 2 a and on fig. 2 b.
Those who have referred to this model often consider mathematics one-dimensionally. It means that they enter just one mathematics "dimension" into each of tnese figures. Usually, they look upon calculation ("arithmetic") as the predominant and necssary topic area of mathematics.

	Mathematical topics						
	a	b	c	d	e	f	etc.
Levels of mathematical complexity 1							
2							
3							
4							
etc.							

Figure 1. The content deviation model.

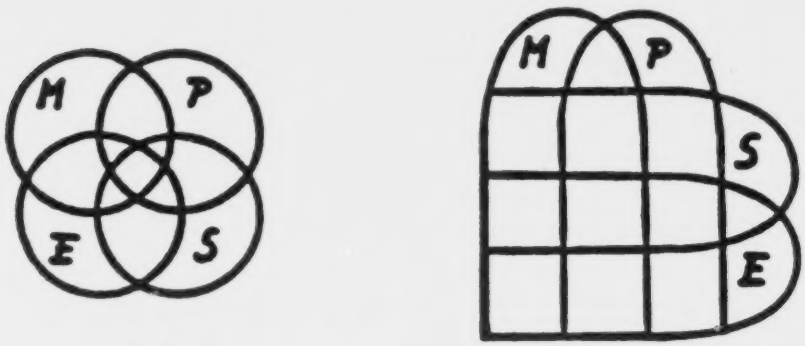


Figure 2. Illustration of the behaviour deviation model.

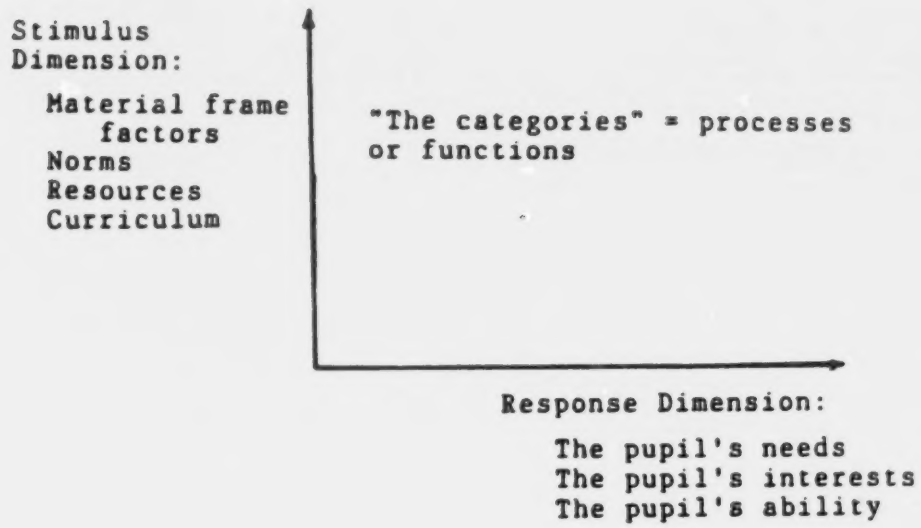


Figure 3. Graph displaying learning process in mathematics education (the factor-interplay model).

The factor-interplay model

Recently, a factor-interplay model has been presented in several works (Uprichard et al. 1975, Nyborg 1986, Magne & Thörn 1987). It is a synthesis of the content deviation model and the behaviour deviation model with simultaneous focus on learning styles of the pupils and the subject matter of mathematics. Research, as well as curriculum innovation and teaching practice, must simultaneously consider both content of mathematics and the pupil's individual reactions.

Functionally, mathematics learning can be represented by a complex vector space with many axes. The main vectors (or factors) belong to the two subspaces of mathematics content (stimulus dimension) and the pupil's ability (response dimension). See fig. 3.

Fig. 4 presents a flow chart to demonstrate consecutive steps of planning according to the factor-interplay model. It should be noted that diagnosis is a simultaneous assessment of the pupil's mathematical performance and of his/her needs, abilities, interests and social and affective inclinations. Prescription refers to the systematic treatment of these two groups of variables to bring about aims of learning.

During diagnosis of curriculum tasks (phase 1) and behaviour diagnosis (phase 2) the pupil's learning problems are studied in order to identify both general behaviour problems and error patterns in mathematics.

Determination of objectives and programmes for the pupil (phases 3 and 4) contain aims for the programmes and prescriptions relating to systematical handling of specified topics.

Implementation (phase 5) and evaluation (phase 6) relate to the pupil's learning in an individual setting and the results of this learning. In phase 6, when the learner has reached one aim of phase 3, work should start on another aim. If an objective is not reached the phase 4 prescription should be revised.

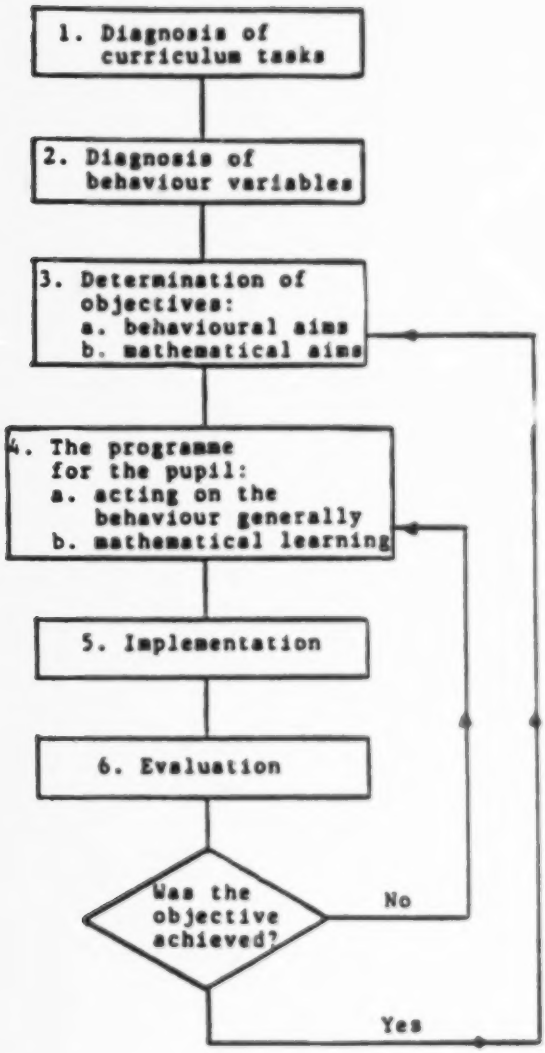


Figure 4. Planning according to the factor-interplay model.

Remarks on the efficiency of remedial teaching

Concluding this survey of remediation, a few words should be added about experiments on teaching dysmathematical pupils in order to improve their mathematics knowledge.

Some early writers reported positive results after remedial teaching in mathematics, for instance Bemis and Trow (1941/42), Guiler and Edwards (1942/43), Tilton (1947). These early investigations have been critised by Rasborg (1961) for various reasons.

With better remediation techniques and more reliable evaluation methods later studies have, however, mainly corroborated the findings that remedial mathematics under favourable conditions leads to considerable progress (e.g. Lytton 1961, 1967), Ljungblad (1966, 1969), Magne (1973, 1974), Schöniger (1989).

In these cases the teaching was done individually or in small groups. These pupils had a normal ability in other school subjects than mathematics. The work was characterised by great intensity of effort from both the teacher and the pupils. Manipulative devises were used when needed in order to develop practical experiences as a basis for conceptual knowledge. Language habits were strongly accentuated. The number of participating pupils totalled from about 25 to more than one hundred subjects in the different experiments. The pupils were 7 to 15 years of age. Two 45-minutes periods every week during one school year were used.

The treatment proved effective for most pupils. The following results may be of interest:

* The overall increase was equivalent to about 1.5 school year.
* The outcome was positive for all levels of intelligence.
* Pupils with high personality test scores benefitted most.
* Persistence, concentration, interest, lack of anxiety and good adjustment at home and in school favoured positive attainment.
* About ten per cent of the pupils did not improve their attainment in relation to their expected development. These pupils were emotionally unstable, hyperactive or less persistent than the average or belonged to problem families.

DIDACTICS IN MATHEMATICS FOR HANDICAPPED STUDENTS

Relations between mathematics and mathematics teaching/learning

We should distinguish between mathematics and mathematics teaching/learning.

Without going into details, it was Frege (1959, 1884) and Russell (1903) who once inspired a formal approach to the philosophy of mathematics and this position is generally accepted as a foundation of mathematical thinking.

Mathematical learning and thinking is not limmited to logical proof. Knowledge of mathematics usually starts with experiences, what sometimes is called every-day activities. Even if one accepts that mathematics is a system of concepts, belonging to a world of formal proofs, it is also a dynamic system where conceptual innovations take place and requires creativity on the part of the learner (cp. Tiles 1984).

Conceptual discoveries and creative actions are parts of mathematical learning. This is something appurtenant not only to the mathematical thinker but also to the learning child. Little by little, the developing individual acquires sensori-motor experiences of a quantitative nature which will eventually become components in such rational thinking that belongs to the Frege-Russell conception of mathematics.

In general terms, the development of mathematical reasoning is the same for all children, including handicapped students. In many cases, however, the handicapped child does not reach the higher levels of conceptual knowledge (cp. Piaget's concret and formal operations).

The findings of our Swedish explorations suggest that mathematical knowledge does not attain the level of conceptual knowledge until a rather late stage of the learning process (Magne 1978).

Through the individual's learning of mathematics runs the connecting thought that the learner must build up experiences which gradually are used in logical reasoning. It is essential to let these experiences complement each other. This means that instruction, only restricted to the "three R's", is incomplete and insufficient. Accomplishment of mathematical

education should have as one "leitmotif" a personality formation principle.

Theories of education are usually attributed to two main categories: associationistic (behaviouristic) or psychodynamic ones.

Such a dichotomisation is not quite consistent. See Juul (1990).

The current constructivism is derived from the psychodynamic theories.

Psychodynamic theories have the common foundation that feelings and needs are of primary importance in the life of the growing child. Some basic needs should be satisfied if the child would develop a healthy personality, for instance need for love, security, attachment, belonging, success, dependence and independence and a personality identity.

Constructivism is the application in mathematics education of a psychodynamic view of life. The following practical consequences are important:

(1) Learning takes place as a social process in everyone's natural environment.

(2) Active learning is a necessary condition for lasting retention. Learning is characterised as a progressive restructuring of relations between self and world where world implies both tangible and representable reality.

(3) Constructivism has many faces. As Lerman (1989, 1990) maintains radical constructivism is a relativist epistemology. According to this view, the progressive acquisition of mathematical conceptions is one which constantly goes on in the mind of the learner.

However, we must find a specified theory for complex learning well suited to mathematics among handicapped. The theory must be adapted to practical situations. Such theories have been studied by Norman (1978) and Nyborg (1971). In his presentation Nyborg stresses the importance of possessing verbal "analysers" in concept learning and shows how language can be used in order to master generalisation and abstraction. Fig. 5 is a revised picture from Norman. It demonstrates a learning sequence taken out of its continuum and how experiences are transformed into mental images, representations and skills.

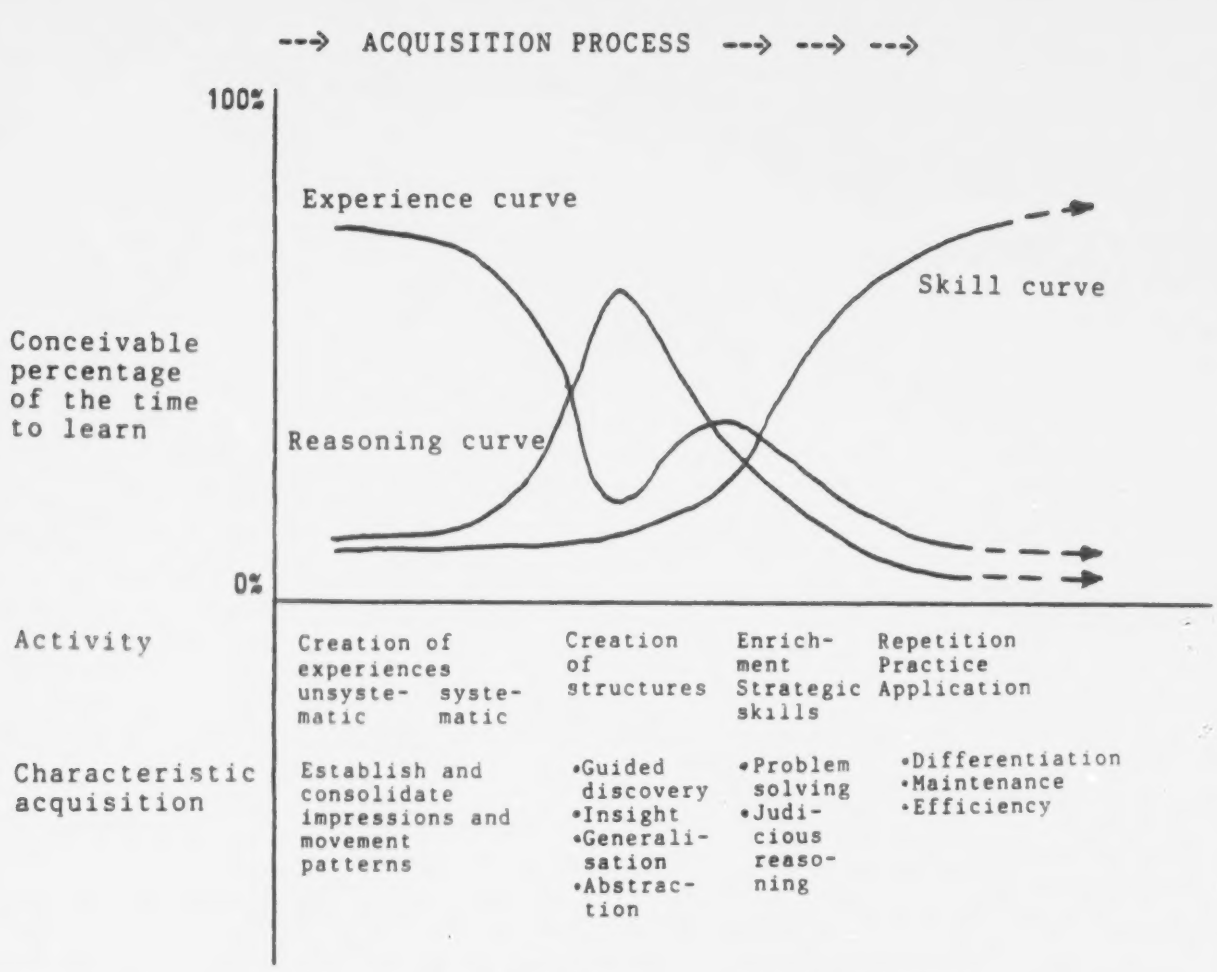


Figure 5. Learning categories related to the development of knowledge categories in mathematics.

Let us assume that there are certain developmental patterns (usually more than one) and that there are certain learning processes (necessarily more than one) which form a learning programme.

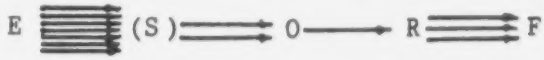
A) Acquisition starts with <u>experiences</u> beginning before the explicit mathematics instruction. At first, these experiences are unrelated but will gradually be systematic and later become related to mathematical events within the present programme.

B-C) As time goes by the child discovers (actively creates) certain invariances (similarities) and, thus, constructs or <u>reconstructs the experiences</u> by conscious speech and gains insights, generalised representations and abstract ideas. Often this occurs by mediation of adults, which means it is a tutorial process.

It is then time for a more advanced type of experience. The freshly discovered structure is used in problem solving and judicious reasoning. In other words, having acquired a reasoning pattern the child learns how to use it. Norman calls such proficiency strategic skills.

D) We characterise the fourth developmental phase of a learning sequence as skill acquisition. The child repeats activities deliberately. Naturally, such repetitions are often incited and made more efficient by parents and teachers.

Constructivism can be illustrated with the expression



The graph may be read as follows: The environment (E) contains many elements of learning. Parents, teachers etc. control the child, single out and point out learning situations (S) and direct the attention of the child (O) to a certain interesting phenomenon which the child cultivates with its creative power (R). The learner continues by unaided efforts, among other things, to put its learning into practice onto new situations (F). Different numbers of arrows indicate ongoing selection as to topics and strategies.

Constructivism also presupposes that the mathematics learning should be balanced, e.g.

balance between main topic areas of the mathematical contents,
balance between methods to learn/teach,
balance between the measures to organise the tutorials or instructional groupings, and
balance between ways to use physical materials.

Thus, the constructivist learning and teaching theory may be divided into the following three obvious components:

(1) The structure of the mathematical matter (stimulus). Constructivism is interpreted as offering the student structured topic units which should be chosen and connected by their inherent logic. The learning is facilitated by optimal emphasis on logical relations and conceptual structure of the whole complex of topics which is presented in a certain programme.

(2) The pupils' active construction of their knowledge of the mathematical subject matter (responses of the pupils). Constructivism is furthermore interpreted as the active work of the pupils to construct (reconstruct or analyse) the structure of the topics and assimilate the topic elements in the individual thinking schemes of each pupil.

(3) The teacher's structural instruction ("catalyst effect"). Finally, constructivism is interpreted as a claim on the commitment of the teacher to promote the active constructive work of the pupil. The teacher should individualise the learning conditions for each pupil in the most favourable degree as to choice of topics, degree of difficulty, learning methods, groupings and material.

The pupil at the centre

One inference of this analysis is that planning for mathematical mathematical instruction must emphasis both learner and subject matter.

Another conclusion is that research should simultaneously discern how individual learners react and how the units of contents can be arranged.

Just as much this does concern curriculum innovation.

To demonstrate this, let us exemplify with research of mathematics instruction for deaf children. Following traditional strategy, planning would have concentrated on contents and production of instructional aids or, alternatively, on the skills of the pupils and, thus, resulted in a biased and restricted view of success or failure in mathematics.

The new cognitive process model leads to another approach. The study of psychological and social behaviour patterns are object for research just as much as content-directed analysis. We have to look for the processes that spring up from the interplay between mathematics content and pupil reactions.

It is possible to formulate a curriculum from the rule that the pupil is at the centre of the education. In that case

* firstly, the mathematical behaviour is mapped out, and
* secondly, those topic areas are priorited which the pupil would need first-hand.

This way of looking at things is particularly fruitful in the education of handicapped pupils.

The graph in fig. 6 illustrates the relations between pupil and school organisation. The figure has the pupil at the top. From the pupil proceed arrows that decide the course of the planning. For a handicapped pupil this concerns for instance

* The adjustment of the education policy and secondarily financing the education of a handicapped individual,
* The learning and teaching methods etc,
* The supply of learning materials, and
* The learning.

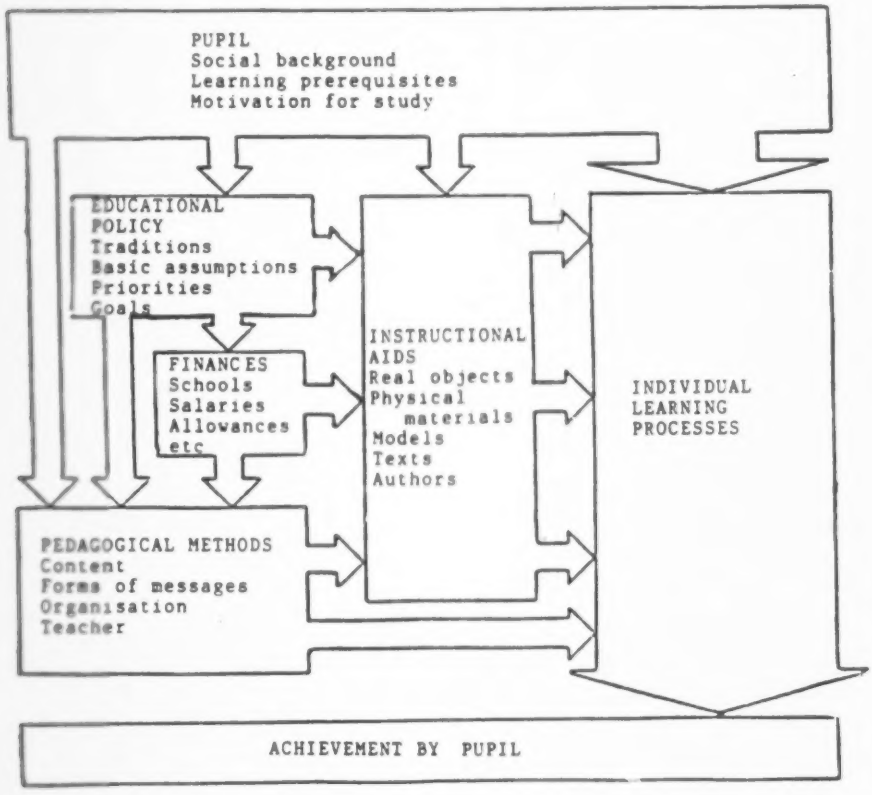


Figure 6. Pupil based educational philosophy (Magne, 1978)

Dysmathematics due to school organisation?

The role of the school organisation for formation and producing learning difficulties in mathematics has only been studied scantily. Some authors have attached some weight to faulty teaching methods, for instance (see Schonell & Schonell 1957):

(a) over-explanation of processes with dull pupils,
(b) over-emphasis of mechanical work,
(c) a 'too-extensive' syllabus,
(d) commencing a new step before the previous one is mastered, and
(e) bad grading of examples.

But the specific importance of general organisational conditions is seldom analysed as regards origin of dysmathematics.

Some of the few investigations on this problem that have been published are Ikäheimo (1889, see Magne 1990b) in Finland and Magne (1990b) in Sweden. They have both followed the achievement in the compulsory school systems in their respective countries from school beginning to school leaving and studied highachievers as well as lowachievers.

With sophisticated assessment procedures they were able to show that particularly the lowachievers solved a gradually smaller amount of examples, typical for each consecutive grade of their nine-years schools.

Thus, the small children in the first grade achieved the greatest proportion of solutions typical for the first grade. The pupils of the ninth grade were least successful of all in the examples that were assessed to be representative for their grade.

As a matter of fact, the mathematical knowledge of the school beginners was surprisingly good. At the end of the first school year the achievement was nearly 100 % in many representative examples. At the other end of the school system representative examples barely reached 60-70 % on an average.

For the lowest deciles of the population the result was disastrous both in Finland and Sweden. There were about 15 % who did not increase their mathematical knowledge after the sixth grade, assessed as their performance measured in absolute numbers.

In many supposedly easy examples the achievement did not exceed 10 % for the lowest 15 % of the school leavers on such examples which were looked upon as typical for the middle grades (for instance to write two thousandths with digits). Highest achievement was registered in the four fundamental operations with natural numbers. Numeration and notation presented surprisingly low percentages in the higher grades (but not in the lower grades). In form perception, measuring and geometry the percentages were low from the beginning and to the end of the school. Problem solving of all sorts and algebra had low frequences in all grades.

This outcome might be interpreted for many pupils as going to school during the three to four highest school years without learning anything at all in mathematics.

It is impossible to give definite reasons for for these results. The similarities between Finland and Sweden imply comparable conditions in these and, possibly, other countries. Hart (1981) has arrived at inferences of the same nature.

Some of Schonell's recommendations should be heeded.

First of all, faulty teaching should be considered a contributary factor in the production of disability in mathematics. What he says about verbal over-explanation and over-emphasis of drill is correct. Much more serious is the ambition of politicians, curriculum makers and textbook writers to burden <u>all</u> pupils with one and the same too extensive syllabus.

Equality should never mean the same set of topics for all. On the contrary, equality must mean opportunities also to develop one's own personality quite independent of others.

This should mean that individual syllabi must be permitted, leading to variable programmes, textbooks, learning aids etc.

No pupil should be required to grapple with complex practical problems absolutely outside one's interest horizon.

The intensity theory

Following this approach it is possible to formulate some general principles in order to individualise the teaching for handicapped pupils:

(1) Personality accentuation,
(2) Intensity approach,
(3) Individualised self-activating learning,
(4) Broad-front strategies or the ecological principle,
(5) Multi-model techniques,
(6) Communication and contact developing strategies, and
(7) Early identification and early intervention.

The intensity theory is an old theory in the sense that its essential features belong to the commonplace, accepted opinion about the educational role of remedial teaching. Nevertheless, it differs in some respects from the conservative view.

The intensity theory recognises some pupils as being so handicapped as to need special tutoring. It permits us to teach the handicapped pupil in an ordinary class, but the crucial aspect is the need of individualised treatment. The programme should be dependent on the diagnosed learning prerequisites and lead to acceleration inside the diagnosed domain. No pupil is assumed to be handicapped in all possible respects. Therefore, the teacher might compensate some of the deficits through intensive instruction and, in some cases, even provide the pupil with psychotherapy during a limited period. This additional teaching may be intensified in several respects: the teacher may choose to teach the pupil in a small group; the strategy might be especially designed and worked out; special teaching aids or particular texts could be used.

The strength of the intensity theory lies in the opportunity it offers for acceleration. It does not exclude the facilities of a natural environment in most activities inside or outside the school, or during the main part of the time. It can easily provide the pupil with effective additional teacher hours and intensified learning periods.

The advantages of the intensity theory are for instance:

(a) Obvious opportunities for acceleration,
(b) Effective chances of energetic expert help,

(c) Input of material and strategy may be maximised temporarily,
(d) Good opportunities to concentrate the maximised learning to critical periods of development,
(e) Takes placement in ordinary class for granted, but accepts various forms of special education,

Play didactics in mathematics for mentally handicapped children

In many countries the aim of the mathematics education obviously is utilitarian, for instance in Sweden:

* To prepare the pupils for their future role as citizens and
* to induce the pupils to acquire a knowledge and skill applicable to other school subjects, future studies, leisure time, and vocation.

It is typical that reference to practical situations is strongly stressed in all mathematics work for pupils in the compulsory school. It is not unusual today to find statements like this, "Nobody uses this type of mathematics, so why drill it and pain the children with it?"

The classical time for "everyday" or "situational maths" was the 1940's when the so-called functional mathematics was discussed. Later came the "new maths" with more sophisticated impulses. During this time Dienes introduced plays and games into mathematics, derived from the thesis of Piaget that the process of forming concepts takes far more time than was earlier believed.

Bartlett, Wertheimer, Bruner and many other researchers have suggested a period of thinking for most children when constructive thinking is the natural way to solve problems. During this period of development play is a normal way to learn mathematics. But also for older pupils, many topics can be studied with the aid of play or games.

It has even been suggested that mathematics has a strong affinity to play.

To exaggerate usefulness in mathematics learning is unsound.

Developmental work in Sweden (Magne 1990a) indicates that play didactics would be useful and efficient in the mathematics education for disabled children as well as for preschool children, particularly in the lower grades of the

special school where age groups 5 to 10 are taught. These explorations are based on the Magne-Thörn (1987) cognitive taxonomy where three main areas are particularly recommended for younger children:

P-area: Language representation and problem solving from a linguistic point of view,

N-area: Numeration and notation, and

G-area: Form perception and space representation, body awareness, money, geometry, measuring and units.

Furthermore, it is assumed that the children during this period of life learn from active, conscious observations and movements. During the first years of life the verbal language seems to be less essential to mathematical learning than perceptual and motor non-verbal behaviour. Concrete experiences are necessary. When language is functionally mastered the language will become a more important motor and, from that time, accelerates the development to abstract reasoning about practical and formal problem solving.

Obviously, mathematical behaviour begins very early (during the first year of life) and this holds true especially for the three main areas referred to.

Play is said to be the foundation of mental growth of the child. In play the children develop perceptual and motor skills. Rules and logic are created during play activities and language habits are being differentiated.

In our Swedish experiments we have found that play should be part of the fundamental activities in the early mathematical learning situations. The aim of such projects on learning by play didactics would cover the following fields of inquiry:

(1) Characteristics and guidelines for studying the processes of mathematical learning of various disabled types of children, for instance mentally handicapped.

(2) The use of various learning aids, including toys and electronics, to help the processes of mathematics learning of mentally handicapped children.

For this purpose a guide book was prepared with suggestions for special educators. The plan presupposes that during the first school years the handicapped children mainly acquire their mathematical experiences by play activities. The learning is so arranged that it forms a structured programme.

The activities follow a spiral disposition. This means that in the beginning easy topics are presented, according to empirical studies. The activities are allocated to three parts, called "spirals". Each spiral contains all three basic mathematical main areas P, N and G.

Naturally, the teacher and the students are free to choose a different order of activities whenever they find this suitable for individualising purposes or for grouping the children according to their interests and ability. The spiral method is illustrated by way of fig. 7.

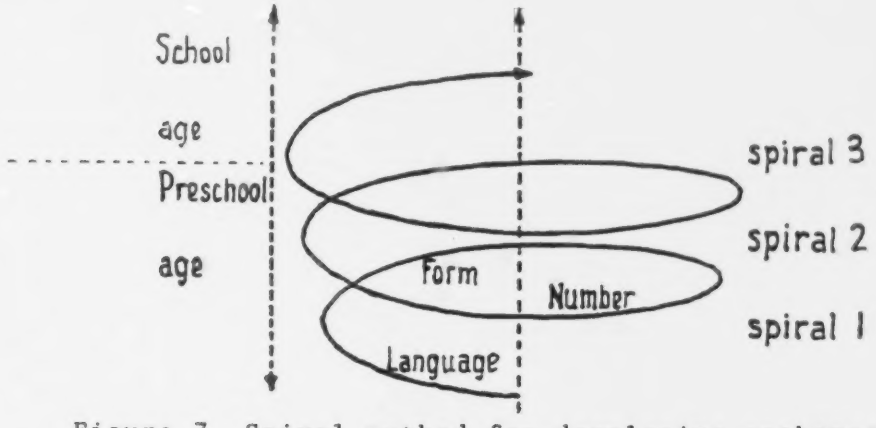


Figure 7. Spiral method for developing mathematical experiences by play didactics.

Reading and writing numbers should be postponed and is suggested to begin, not earlier than in connection with the third spiral for the disabled pupils. This does not exclude opportunities to be acquainted with written or printed material and work with such material. There are many suggestions in the Guide Book about individual block-construction, drawing, painting etc.

The selection of activities has particularly been influenced by research by the Agam Project (1990), Frostig & Horne (1964), Frostig & Maslow (1970), Gran (1982), Magne

(1988b, 1990a), Nyborg (1971), Ostad (1982), and Piaget & Inhelder (1977).

The Guide Book contains the following four chapters:

1. Mathematics and play
2. Spiral 1
3. Spiral 2
4. Spiral 3.

Spiral 1.

Introduction on the activities.

The Letters P, G and N refer to the above mentioned main areas. The numbers stand for themes within each main area.

P 1 The box of animal toys
P 2 Big - small, biggest - smallest, bigger/smaller than
P 4 Breadth - width
G 13 Formboard and puzzles
G 25 Movement and rythm (from Frostig)
G 23 Drawing - painting - modelling
G 4 Attention
G 26 Round things (from Agam)
G 27 Square-shaped things (from Agam)
G 5 Hidden objects - hidden figures
N 2 Counting objects by pointing/moving
N 5 Finding differences in a set of objects
N 6 As many as - the same number
N 7 Few - fewer
N 11 About sets of three objects.

Spiral 2.

Introduction on the activities
P 3 More about size
P 6 High and tall - short and low
P 7 Thickness
P 8 Height, length and width
P 9 Age
P 10 Order
P 11 Likeness and unlikeness - belonging together
P 12 Sharing - partitioning
P 13 Giving another child a present
G 1 Balance - body control
G 28 Three-cornered objects (from Agam)
G 29 Vertical direction (from Agam)
G 30 Horisontal direction (from Agam)
G 31 Neither vertical nor vertical direction (from Agam)
G 2 Parts of the body - right and left
G 6 Draw a man
G 15 Build just as high - just as long - just as wide
G 7 Draw a house - a hill - a water glass
N 3 Point and count
N 13 Build a staircase
N 9 Sets of three objects

N 10 More activities about the number three
N 14 Sets of four objects
N 15 Sets of two objects
N 19 Dot patterns representing numbers
N 22 Span of apprehension ("subitizing")

Spiral 3.

Introduction on the activities
P 5 Weight and mass
P 14 When you get more things than you had before
P 15 Conservation of length, volume and area (from Piaget)
P 16 Number stories
P 17 Mathematics and gymnastics
G 34 Money
G 32 Straight and curved strokes (from Agam)
G 11 The daily dose of time
G 12 How to use a balance
G 8 How to copy figures
G 9 Drawing with a template
G 10 Look out for reversals
G 16 Draw the same figure in a net of dots
G 17 Name the blocks
G 14 Play catch
G 18 Play ball with a racket
G 19 Skipping
G 20 Paper folding
G 21 A purse as a paper-folding activity
G 22 Cardboard models
G 24 Things which float and sink in water
N 25 Numbers 1 to 5 in concrete situations
N 21 Conservation of number (from Piaget)
N 16 Sets of five objects
N 17 The number one
N 18 Nothing - nought - zero - nil - (cipher)
N 12 Order - some ordinal numbers
N 20 Ordinal numbers - counting backwards 5 - 1
N 4 Numbers in sports
N 25 Dice-playing
N 1 Different methods to count
N 8 Checking up on your counting
N 23 Advanced ways of counting
N 24 Counting two by two

Each activity section offers activities belonging to a definite theme. The standard type of an activity section in the Guide Book runs as follows.

(1) A situation is suggested. This means the following. The teacher and the children arrange together the acting and parts of the children and collect the necessary materials. They talk about the situations they meet.

(2) Situations on related themes are suggested.
(3) Games or alternative play to follow up the theme.
(4) Songs, nursery rhymes, fairy-tales etc.

In 1987 a class of student preschool teachers participated in a study of play activities in mathematics in order to validate the Guide Book. The children were 3 to 6 years of age. They all went to local preschools in the city of Malmö. Summarising the results, the activities were considered popular among the children, as well as among the teachers in the preschools and among the preschool student teachers. The experiment also indicated that the preschool children extended their knowledge of mathematical topics. The conclusion was that systematic and structured exercises according to the play activity method would be effective in mathematics.

Perspectives on the future

Traditional mathematics education apparently has proved unsuccessful for many handicapped pupils. New didactics, among others according to constructive theory, have been worked out. These new methods seem to suit some children. Other children require alternative approaches. But the value of these remediation techniques awaits further empirical validation. Additional research is much needed.

Disabilities connected with mathematical learning still represents a neglected area of research. There are many studies on reading problems but comparatively little research on low achievement in mathematics. This state of affairs seems astonishing as a quite large number of pupils manifest difficulties in their learning mathematics. In the United States some authors indicate higher incidence of arithmetic disabilities than in reading/spelling. See for instance Badian (1983), Fletcher, Satz & Morris (1984).

There are many problem fields which should no doubt be closely examined. Some of the following issues have been touched upon in this chapter.

* Handicapped children present great heterogeneity within groups of similar impairment as well as between groups of different impairments. What are the essential prerequisites to learn various mathematical topics?
* Mathematical reasoning has been described as mainly abstract but, at the same time, is a means to understanding the real world. Certainly, mathematics is often more known for its applications than its abstract nature. But how can we guide a handicapped child from totally concrete experiences to purely mental operations inherent in the mathematical formal reasoning or problem solving?
* The importance of preschool mathematics development to later mathematical achievement is obvious. But how can early cognitive experiences be used for further acquisition of knowledge?
* Perceiving difficulty always is concomitant with mathematical learning. But how can emotions and stress become positive learning factors rather than lead to delusion and frustration?
* Most textbooks are adapted for homogeneous ability grouping. How should instructional texts be constructed for integrated education of heterogeneous groups of handicapped pupils in ordinary classes or in remedial classes?
* Mainly lacking is work on theories for mathematical remediation, for instance coordination of mathematical, educational, psychological and medical approaches. How can we get a more comprehensive understanding of conditions and causes of learning difficulties in mathematics?

CONCLUSION

One problem today is that much traditional arithmetic is obsolete. Future need of mathematics has not always use of school arithmetic of yesterday. The handicapped citizen of tomorrow should have at least an inkling of a more useful but also a more imaginative mathematics and less drill of calculations in a textbook.

But more important, let us remember that we are talking about the happiness of thousands, perhaps millions of children in many countries. We are talking of young people who may be helped to a wealth of satisfaction if they might succeed in their studies and choice of a profession. We are talking about happy school beginners who may soon feel dispair at their constant failures, or confidence in their acquired experience and understanding. We are talking about children who try to get some mathematical literacy and its never-to-be overestimated by-product of reasoning ability. They will be able to appreciate their skillfulness at the same time as they reach a little bit of this literacy.

No person should remember his schooldays as the part of his life when he learned nothing. Instead we ought to aim at giving all pupils as satisfying impressions from their schooling as is possible.

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Abstract card

Magne, O. Dysmathematics. Facts and theories concerning mathematics learning for handicapped pupils. Educational and Psychological Interactions (Malmö, Sweden: School of Education), No. 106, 1991

Ranschburg, Hungary introduced, in 1905, studies of disability to learn mathematics. In 1920 studies were published on brain localisation of "acalculia". The author suggests dysmathematics as a name for these conditions. Psychologically and medically dysmathematics appears as a set of complex and multi-factored personality dysfunctions. Remediation seems to be best furthered by the factor-interplay model. But it is a great need for more intensified research. Play didactics is suggested to be used more in school practice than is fashionable today.

Keywords: Constructivism, disability, dysmathematics, error pattern, handicap, mathematics learning, play didactics, remedial teaching.

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